nism. Fibers made by different procedures are now being compared.

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Tsuyoshi Ohnishi

Department of Physics, Faculty of Science,
Nagoya University, Nagoya, Japan

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A Suggested Technique for Measuring Stress Relaxation Modulus and Creep Compliance and for Testing Linear Viscoelastic Theory

Stress relaxation is measured by subjecting a specimen at time zero to a suddenly applied strain, ϵ, which is thereafter held constant. The stress, σ, is measured as a function of time, and the quotient $\sigma(t)/\epsilon$ is the relaxation modulus, $G(t)$. Creep is measured by subjecting a specimen at time zero to a suddenly applied stress, σ, which is thereafter held constant. The strain, ϵ, is measured as a function of time, and the quotient $\epsilon(t)/\sigma$ is the creep compliance, $J(t)$.

In a stress relaxation experiment, it is difficult to measure the stress in the specimen without changing the strain, and most methods to date accept this disturbance but try to keep it negligibly small. In this letter, we shall show how some relations in the theory of linear viscoelasticity may be used to give the relaxation modulus and the creep compliance from data taken from a mixed system, by which we mean a system in which neither the stress nor the strain is constant.

Theory and Suggested Technique

In a linear viscoelastic material, the Laplace transforms of the stress and strain may be related through the transforms of either the relaxation modulus or the creep compliance, as follows:[1]

$$L\sigma(t) = sLG(t)L\epsilon(t) \quad (1)$$

$$L\epsilon(t) = sL\sigma(t)LJ(t) \quad (2)$$

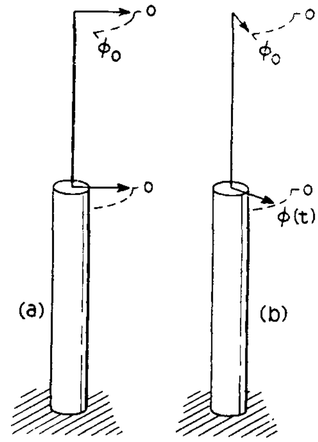

Fig. 1. (*a*) The unstressed system at $t < 0$. (*b*) The system at $t > 0$. The deformation φ_0 is held constant, and the other, $\varphi(t)$, is observed as a function of time.

where L indicates the transform. Also

$$LG(t)LJ(t) = 1/s^2 \quad (3)$$

Now let us consider the arrangement of Figure 1. A torsion wire of known relaxation modulus $G_1(t)$ is attached to the specimen whose unknown relaxation modulus $G_2(t)$ is desired. (It will be of advantage if the wire is as nearly perfectly elastic as possible, when $G_1(t)$ will be nearly constant.) The relaxation torsional stiffness, $M(t)$, of the wire of radius r is

$$M(t) = G_1(t)K_1/A_1$$

where

$$K = \pi r^4/2$$

and A denotes length. Similarly, that of the specimen is

$$N(t) = G_2(t)K_2/A_2$$

Then eq. (1) becomes

$$LT(t) = sLM(t)L\varphi(t)$$

where T is torque and φ is the twist in radians.

Before time zero, $\varphi_0 = \varphi = 0$, and the system is stress-free. At time $t = 0$, the upper end of the wire is suddenly twisted to φ_0, as a result of which the junction between the wire and the specimen starts a wandering angular history $\varphi(t)$. This is the quantity to be observed during the experiment. The twist in the wire is, then, $\varphi_0 - \varphi(t)$. The torque is the same in both wire and specimen, and its transform is, for the wire,

$$LT(t) = sLM(t)L[\varphi_0 - \varphi(t)] \quad (4a)$$

and for the specimen,

$$LT(t) = sLN(t)L\varphi(t) \quad (4b)$$

whence

$$LM(t)L[\varphi_0 - \varphi(t)] = LN(t)L\varphi(t) \quad (5)$$

whose inverse is

$$\int_0^t M(t - \tau)[\varphi_0 - \varphi(\tau)]d\tau = \int N(\tau)\varphi(t - \tau)d\tau \quad (6)$$

All the functions in the integrands are known except $N(\tau)$. The integral on the left may therefore be evaluated numerically as a function of t, after which the integral on the right may be evaluated for $N(\tau)^2$ when $G_2(t)$ is merely $A_2N_2(t)/K_2$. If $J(t)$ also is wanted, it may be found by a similar numerical solution of the inverse of eq. (3),

$$\int_0^t J(\tau)G(t - \tau)d\tau = t$$

Suppose the wire relaxes very slowly, as compared with the specimen. If $M(t) \gg N(t)$, the wire will be very stiff, $\varphi(t) \approx \varphi_0$, and the situation approximates that of the classic stress relaxation experiment. If, on the other hand, $M(t) \ll N(t)$, $[\varphi_0 - \varphi(t)]$ will be large in relation to $\varphi(t)$, small changes in $\varphi(t)$ will not change the stress very much, and the classic creep experiment is approached. Thus by changing $M(t)$ we may cover a range of experiments from nearly pure relaxation to nearly pure creep, from all of which we should derive the same function $N(t)$. This, then, provides a way of determining just how good the linear viscoelastic theory is for the given material and conditions.

The technique is not limited to torsion, but may be used in tension, compression, simple shear, or bending. The essential thing is that a fixed deformation (in the torsional case, φ_0) be shared between the known and the unknown substances, with the movement of the unrestrained junction between them suitably measurable.

In cases where the linear theory is applicable, eq. (6) can be used to predict movement in apparatus assemblies of two materials whose relaxation characteristics are known. In this case, $\varphi(t)$ is the unknown function that is sought. Also, the relaxation of stress could be found by using eqs. (4a) or (5b). More complicated assemblies can be treated, with corresponding increase in complexity but no difference in principle.

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I. L. Hopkins

Bell Telephone Laboratories, Inc.
Murray Hill, New Jersey

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